Estimating basis functions for spectral sensitivity of digital cameras

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Abstract Spectral sensitivity of digital cameras plays an important role for many computer vision applications. However, less attention has been drawn on estimating the spectral sensitivity of commercial cameras, and there is neither comprehensive analysis of those spectral characteristics. This paper investigates the characteristics by extracting the basis functions of them by using SVD (Singular Value Decomposition); we have collected data from the literature but also by measuring the sensitivity of different cameras. This paper compares the extracted basis functions with different mathematical basis functions and obtains the optimum set of basis functions. The extracted basis functions can be used to estimate the unknown spectral sensitivity of an arbitrary camera.

Key words

1. Introduction

Spectral sensitivity of digital cameras is non-trivial information for many computer vision applications. Different cameras usually produce differently-colored images for the same scene, regardless of how well adjusted the white balance is, due to the difference in the spectral sensitivity. When the spectral sensitivities of those cameras are known, color of one camera can be converted into that of the other. This would help a number of applications based on colors such as object recognition, object detection, image retrieval, etc. Several methods of physics-based vision also require spectral sensitivity.

While much attention has been paid for camera response-curve estimation [1], less has been drawn on estimating the spectral sensitivity. A common way to obtain the spectral sensitivity is to use a monochrometer [2]; spectral response of a camera can be measured by taking images of a light whose wavelength is tuned by a monochrometer. Hardeberg et al. proposed a method that estimates spectral sensitivity by inverting the system of linear equations obtained by image intensities and known spectral reflectances [3], while it has not been applied to real data because of the instability.

Regarding the analysis of natural spectra, a number of studies have been investigated. Judd et al. [4] and Slater et al. [5] have analyzed the basis functions of outdoor illumination spectra; both of them concluded that the first three bases dominantly covers the entire spcetral distributions. Several researchers have analyzed the reflectance of Munsell color chips and extracted the first four to eight basis functions [6] [7].

This paper investigates basis functions for spectral sensitivity of cameras. In order to extract basis functions, the SVD (Singular Value Decomposition) is performed to the data that are collected from the literature and our experiments. The extracted set of basis functions are compared to mathematical basis functions by applying them to recover the unknown spectral sensitivity from the set of image intensities and spectra.

The rest of the paper is organized as follows: Section 2 describes the benefits of using basis functions for spectral sensitivity estimation. Section 3 introduces several basis functions that should be suitable for spectral sensitivity estimation. Experiments and the results are explained in Section 4, and we conclude our paper in Section 5.

2. Benefits of Using Basis Functions for Spectral Sensitivity Estimation

Basis functions reduces the dimension of spectral sensitivity, because the number of basis functions required are much less than the dimension of sensitivity itself. This reduces the number of unknowns in estimating sensitivity, and thus it provides more accurate results.

2.1 Image Formation

The image intensity is related to the incoming spectrum and the spectral sensitivity of a camera. Concretely, it can be described as

$$I_c = \int L(\lambda) q_c(\lambda) d\lambda, \qquad (1)$$

where $L(\lambda)$ is the incoming spectrum, $q_c(\lambda)$ and I_c are the spectral sensitivity and the image intensity for R, G and B channels. The index c stands for R, G and B.

If we discretize Equation (1), then it becomes

$$I_c = \sum_{\lambda=1}^{W} L_\lambda q_{c\lambda}. \tag{2}$$

where λ is the index, W is the total number of elements, L_λ and $q_{c\lambda}$ are sampled values of $L(\lambda)/d\lambda$ and $q_c(\lambda)$, and $d\lambda$ is the sampling interval.

2. 2 Recovering Spectral Sensitivity

If we use a vector notation to Equation (2), it can be converted to

$$I_c = [L_1, \cdots, L_W][q_{c1}, \cdots, q_{cW}]^t. \tag{3}$$

Let us suppose that we have a set of incoming spectra and corresponding image intensities. Then, Equation (3) becomes as follows by using a matrix notation:

$$\mathbf{I} = \mathbf{LQ} \tag{4}$$

where $\mathbf{I}$ is an $N \times 1$ matrix of image intensities (N is the number of different images), $\mathbf{L}$ is an $N \times W$ matrix of spectra (W is the number of samplings), and $\mathbf{Q}$ is an $W \times 1$ matrix of spectral sensitivity.

When $\mathbf{I}$ and $\mathbf{L}$ are known, $\mathbf{Q}$ can be solved as follows:

$$\mathbf{Q} = \mathbf{L}^{+}\mathbf{I} \tag{5}$$

where $\mathbf{L}^{+}$ is the psuedo inverse of $\mathbf{L}$ and is equal to $(\mathbf{L}^t\mathbf{L})^{-1}\mathbf{L}^t$.

Here, the size of the matrix $\mathbf{L}^t\mathbf{L}$ is $W \times W$. In order to calculate the inverse matrix of $\mathbf{L}^t\mathbf{L}$ robustly, its rank should be W. However, the rank of $\mathbf{L}$ is at most N when N is smaller than W. This happens when the number of samplings is more than the number of images. Then, the calculation of the inverse matrix becomes unstable.

2. 3 Benefits of Using Basis Functions

Spectral sensitivity can be robustly estimated from Equation (4) by using the basis functions of spectral sensitivity owing to its low dimensionality. Let us assume that the spectral sensitivity can be approximated by a linear combination of a small number of basis functions:

$$q(\lambda) = \sum_{i=1}^{D} q_i Q_i(\lambda) \tag{6}$$

where D is the number of basis functions, q_i is the coefficient and $Q_i(\lambda)$ is the basis function.

By substituting the equation into Equation (1), we can derive

$$\begin{aligned} R &= \int L(\lambda) \sum_{i=1}^{D} (q_i Q_i(\lambda)) d\lambda \\ &= \sum_{i=1}^{D} q_i \sum_{\lambda=1}^{W} L(\lambda) Q_i(\lambda) \end{aligned} \tag{7}$$

where R is the image intensity for the red channel.

If we use another notation E_i to describe the multiplication of spectrum data and basis function of spectral sensitivity, namely,

$$E_i = \sum_{\lambda=1}^{W} L(\lambda) Q_i(\lambda), \tag{8}$$

then by substituting Equation (8) into (7), we obtain

$$R = \sum_{i=1}^{D} q_i E_i. \tag{9}$$

The same equations for blue and green channels can be obtained in the same manner.

Now, let us suppose that we have N set of data (image intensities and spectra). By using the matrix notation, we can describe Equation (9) as

$$\mathbf{I} = \mathbf{Eq}, \tag{10}$$

where I is the $N \times 1$ matrix, E is the $N \times D$ matrix, and $\mathbf{q}$ is the $D \times 1$ coefficient matrix. Consequently, this coefficient matrix $\mathbf{q}$ can be expressed as

$$\mathbf{q} = \mathbf{E}^{+}\mathbf{I} \tag{11}$$

where $\mathbf{E}^{+}$ is the pseudo inverse of the matrix $\mathbf{E}$.

If the rank of the matrix $\mathbf{E}$ is bigger than D, namely, if the number of images N is bigger than the number of dimension D, we can robustly estimate a unique solution of coefficient matrix $\mathbf{q}$. Then, we can correctly recover the spectral sensitivity.

3. Optimum Basis Functions

In order to find the optimum basis functions, we tried four different kinds of basis functions to describe the spectral sensitivity, which includes polynomials basis, fourier series, radial basis functions (RBF), and basis functions calculated from singular value decomposition (SVD).

3. 1 Polynomial Basis

Polynomial basis function is expressed as:

$$F = \sum_{i=0}^{D} a_i \lambda^i \tag{12}$$

where a_i is the coefficient. Using this polynomial basis functions, the spectral sensitivity is describe as a linear combination of λ^i (the value of i is from 0 to D) as shown in Equation (8). The Fig.1 shows the polynomial basis functions. In Fig.1, eight basis functions are shown with different colors.

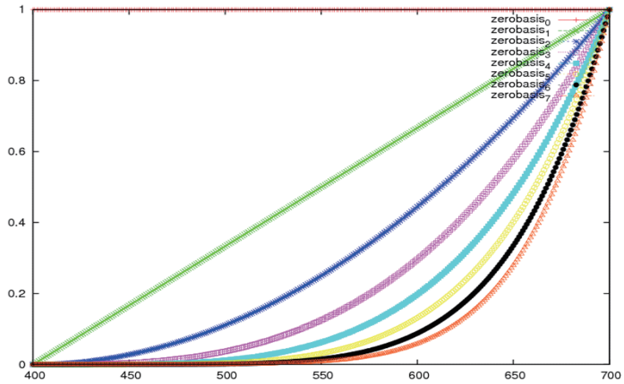

Fig. 1 Polynomial basis functions.

3.2 Fourier Series

The basis functions of fourier series is described as:

$$F = \sum_{i=0}^{D} a_i sin(i\lambda\pi) \quad (13)$$

where a_i is the coefficient. The Fig.2 shows the first four fourier basis functions, the Fig.3 shows the other four fourier basis functions.

3.3 Radial Basis Functions

By using Radial basis functions, the spectral sensitivity is represented as a sum of D radial basis functions, each associated with a different center μ, and weighted by an appropriate coefficient σ. The radial basis functions is written as:

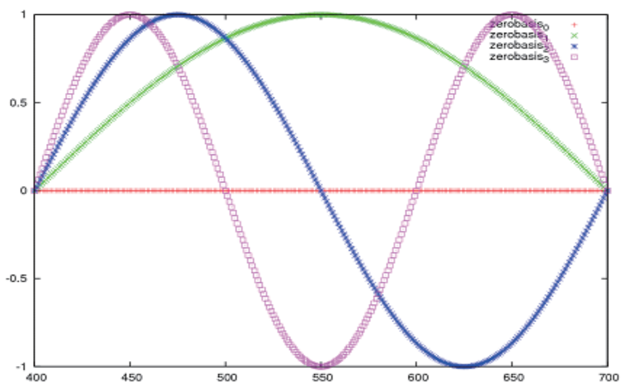

Fig. 2 The first four fourier basis functions.

$$F = \sum_{i=0}^{D} a_i \exp(-\frac{(\lambda - \mu_i)^2}{\sigma^2}) \quad (14)$$

The Figs. 4, 5 and 6 show the RBF basis functions for RGB channel respectively.

The advantage of using the RBF basis function is that these basis functions are similar to Gaussian function, also similar to the shape of spectral sensitivity. Hence, better results can be obtained by using the RBF basis function.

3.4 Basis Functions from Singular Value Decomposition

We collected several cameras and estimated the spectral sensitivity for these digital cameras. Also we collected a few estimated spectral sensitivity from the literature. Then we made a database of spectral sensitivity.

By applying the singular value decomposition (SVD) for the database, we can calculate the eigenvectors and use these eigenvectors as the basis functions to estimate the spectral sensitivity of an arbitrary camera. Details of obtaining the sensitivity database and estimating the basis functions from the database are explained in the next section.

4. Experiments

This section shows the results of estimated spectral sensitivities by using four basis functions described in the previous section. By comparing the results obtained with different basis functions, we find the optimum set of basis functions which has the least error to estimate the spectral sensitivity for an arbitrary camera.

4.1 Obtaining Sensitivity Database

In the following experiment, we use the white board which is illuminated by the monochrometer. The image intensities and spectra of the white board are simultaneously captured by cameras and a spectrometer

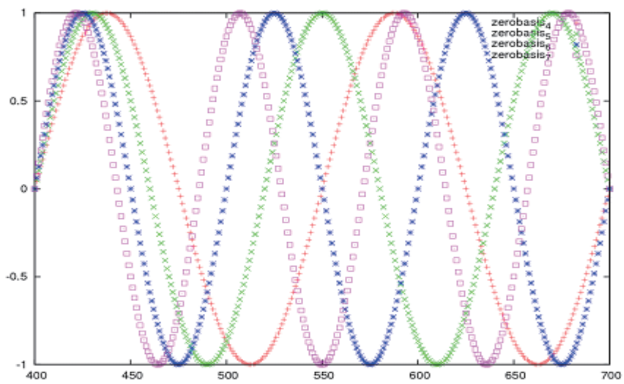

Fig. 3 The Last four fourier basis functions.

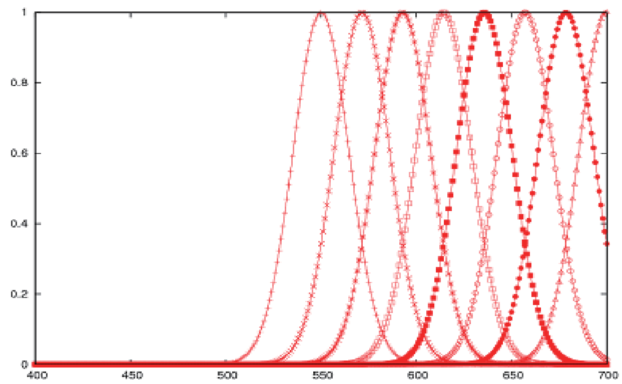

Fig. 4 Red channel of RBF basis functions.

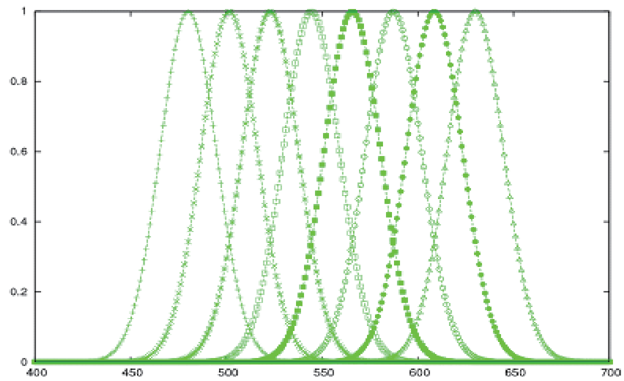

Fig. 5 Green channel of RBF basis functions.

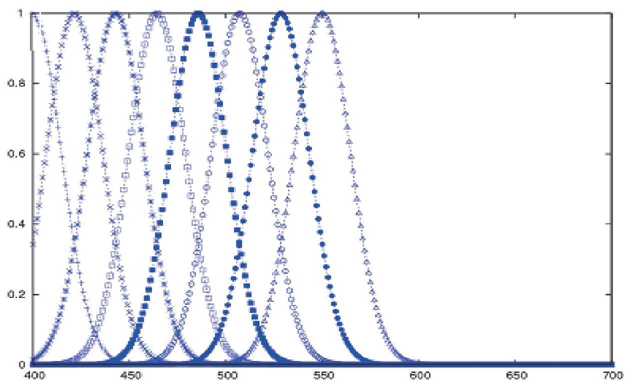

Fig. 6 Blue channel of RBF basis functions.

respectively. The spectral sensitivity is expressed as:

$$S(\lambda) = \frac{I(\lambda)}{n \cdot \int L(\lambda) d\lambda} \tag{15}$$

where $S(\lambda)$ is the spectral sensitivity, $I(\lambda)$ is the image intensity, $L(\lambda)$ is the spectrum, and n is a factor related to the camera aperture, the exposure time and the electronic amplification (the ISO number).

For red channel of spectral sensitivity, we write the equation as shown in Equation (16), where N denotes the number of images taken by a camera. The equations for blue and green channels can be obtained in the same way. Image intensity is read from captured image, and spectrum data are measured by a spectrometer. Thus, the spectral sensitivity is calculated from the above equation.

$$\begin{bmatrix} I_{R1} \\ I_{R2} \\ \vdots \\ I_{RN} \end{bmatrix} = \begin{bmatrix} L_1 \cdot S_{R1} \\ L_2 \cdot S_{R2} \\ \vdots \\ L_N \cdot S_{RN} \end{bmatrix} \tag{16}$$

a) Intensity Linearization

Before using the image intensity which is read from images to calculate the spectral sensitivity, we must linearize it first. Because for most cameras, the response function is not a linear function. For estimating the response function of these cameras, we use the method proposed by Takamatsu et al. [1]. This method is based on probabilistic intensity similarity measure which is the likelihood of two intensity observations corresponding to the same scene radiance. It requires a few images of a static scene taken from the same viewing position with fixed camera parameters. We took a few images of a macbeth color chart for collected cameras to estimate the response function.

b) Intensity Normalization

When taking the images of a white board, according to different wavelength of the light spectrum, the camera parameters (n in Equation (15)) are changed. In order to calculate the spectral sensitivity using Equation (16), the image intensity have to be normalized. The normalization factor n in Equation (15) is expressed as:

$$n = \frac{ISO \cdot t}{F^2} \tag{17}$$

where ISO is the value of electronic amplification, t stands for the exposure time, and F means the f number of each image.

c) Spectrum Measurement

The spectrum data of the white board are measured by a spectrometer, Photo Research PR-655.

d) Estimated Spectral Sensitivity

We collected a few cameras, and estimated the spectral sensitivity by using Equation (16). We also obtained several results of spectral sensitivity from the literature. All the spectral sensitivity are added into a database, then we apply the singular value decomposition (SVD) method for the database to extract the basis functions. The spectral sensitivity of SONY DXC 9000, Nikon D70 and Canon 10D are shown in Figs. 7 and 8.

The spectral sensitivity of SONY DXC 9000 is obtained by ourselves, and the estimated spectral sensitivity is used as the ground truth for the evaluation experiment.

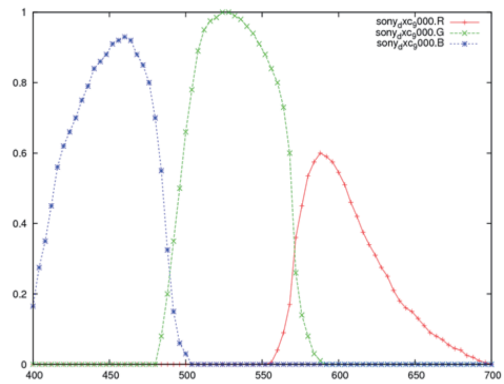

Fig. 7 Spectral sensitivity of SONY DXC 9000.

Tbl. 1 Percentage of Each EigenValue

EigenValues	Percentage
5.574994	76.3%
0.788019	10.8%
0.428096	5.9%
0.261325	3.6%
0.137284	1.9%
0.115380	1.6%

The spectral sensitivity of Nikon D70 and Canon 10D are collected from the literature.

e) Result

From the database of spectral sensitivity, we compute the eigenvalues by SVD. The result of red channel is shown in Table 1.

From this table we see that for the first four eigenvalues the sum of their percentage is 97%. This means that we can take the corresponding eigenvectors to cover 97% information of spectral sensitivity.

Based on the analysis of eigenvalues, the number of basis functions can be decided. Then we obtain basis functions by extracting the eigenvectors of the spectral sensitivity database. The result is shown in Fig. 9.

These Figs. 9, 10 and 11 show the R, G and B channel of estimated basis functions by SVD from the spectral sensitivity database respectively. Not all the basis functions are shown here.

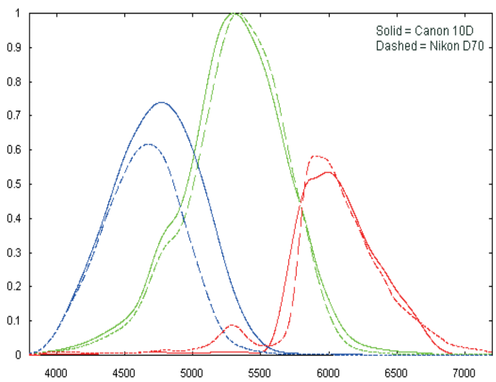

Fig. 8 Spectral sensitivity of Nikon D70 and Canon 10D.

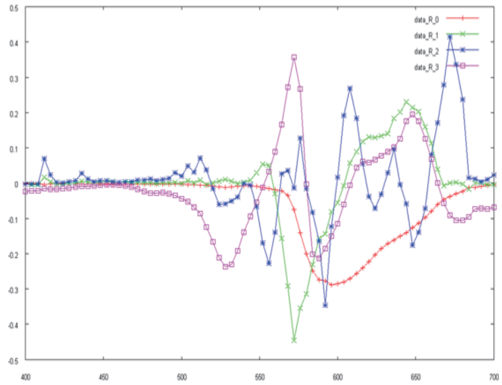

Fig. 9 Extracted basis function of red channel from SVD.

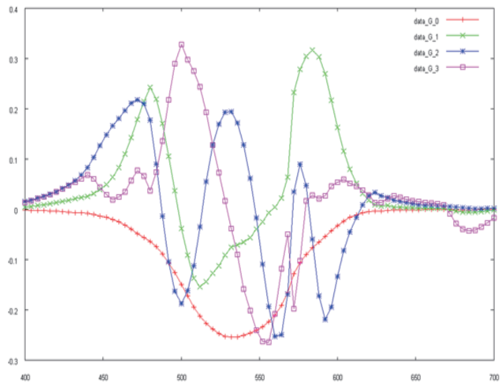

Fig. 10 Extracted basis function of green channel from SVD.

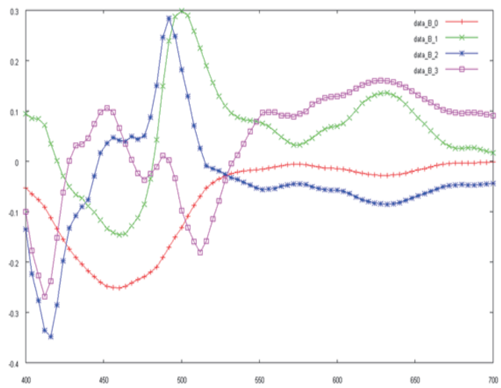

Fig. 11 Extracted basis function of blue channel from SVD.

4.2 Evaluation of Optimum Basis Functions

As shown in Equation (11), the coefficient matrix is calculated from image intensity and spectrum data. Then we estimate the spectral sensitivity by multiplying the coefficient by corresponding basis functions. In order to evaluate the optimum basis functions which has least error, we did the experiment with four different kinds of basis functions. The result of these basis functions are shown in Figs. 12, 13, 14 and 15.

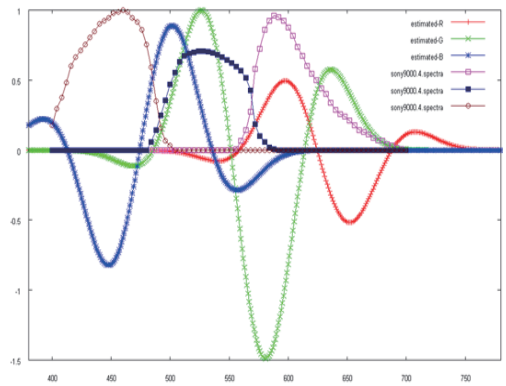

Fig. 12 Spectral sensitivity estimated from polynomial basis functions.

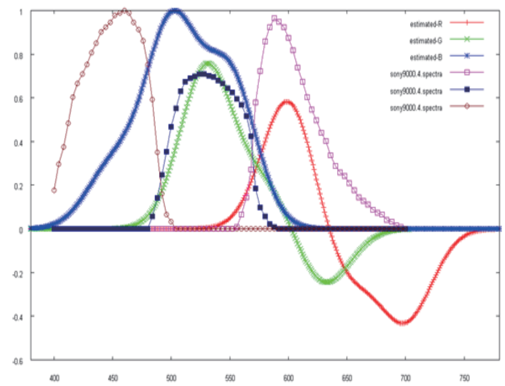

Fig. 13 Spectral sensitivity estimated from fourier basis functions.

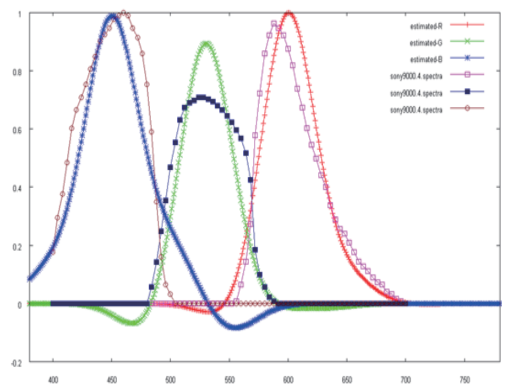

Fig. 14 Spectral sensitivity estimated from radial basis functions.

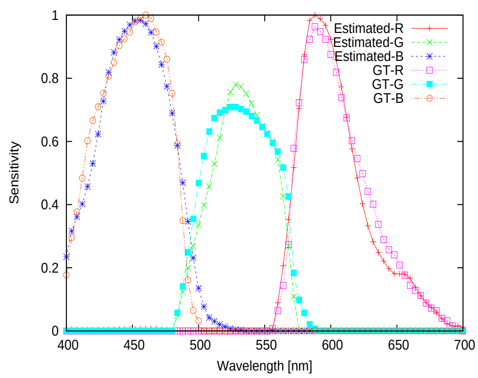

Fig. 15 Spectral sensitivity estimated from singular value decomposition.

From these results, we see that the estimated spectral sensitivity of the radial basis functions is the best, and that of polynomial bases is the worst, as expected. The extracted basis functions by singular value decomposition may not be sufficiently accurate for estimating the spectral sensitivity. This should be because the small number of spectral sensitivities obtained in the database. Another reason should be that the images used for estimating spectral sensitivity had too much noise.

5. Conclusion

In this paper, we have analyzed the characteristic of spectral sensitivity. We obtained the spectral sensitivity by estimating the collected cameras and from the literature. Then we added all these spectral sensitivity of different digital cameras to a database and extracted the eigenvectors by using the singular value decomposition. We compared the extracted basis functions by (SVD) with three other basis functions, polynomial basis and fourier series and radial basis function (RBF) to get the optimum basis function. Based on the experiment, we found that the radial basis functions (RBF) are much more suitable for estimating the basis functions.

For the future work, we will add more spectral sensitivity of digital cameras to the database to obtain much more accurate basis functions. And for the radial basis functions we will improve the algorithm to make it automatically calculate the optimum parameters.

謝辞

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